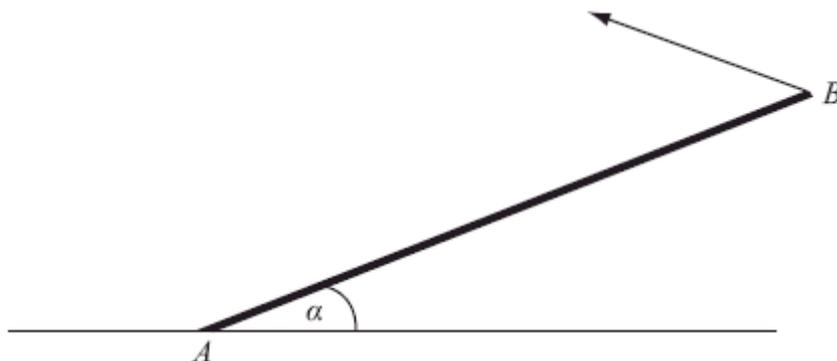


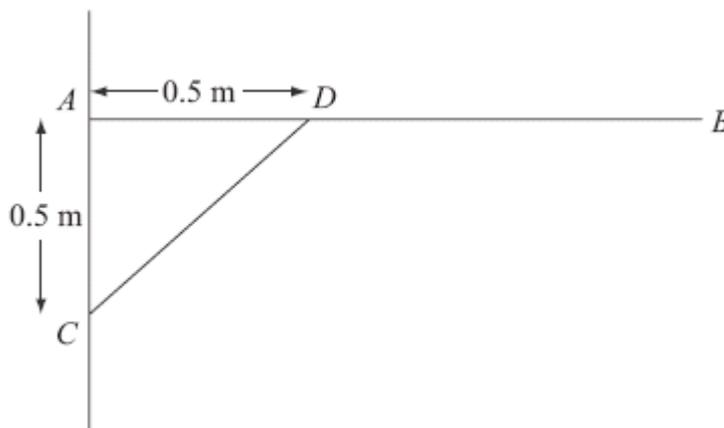
1.



A uniform rod AB , of mass 20 kg and length 4 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a force acting at B , as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at A .

(Total 7 marks)

2.



A uniform rod AB , of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at A . The rod is held in equilibrium in a horizontal position by a light strut CD as shown in the diagram above. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end C of the strut is freely jointed to the wall at a point 0.5 m vertically below A . The end D is freely jointed to the rod so that AD is 0.5 m.

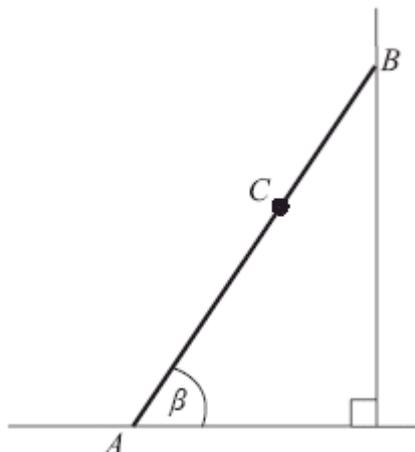
(a) Find the thrust in CD .

(4)

- (b) Find the magnitude and direction of the force exerted on the rod AB at A .

(7)
(Total 11 marks)

3.



The diagram above shows a ladder AB , of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

- (a) Find the magnitude of the frictional force of the ground on the ladder.

(3)

- (b) Find, to the nearest degree, the value of β .

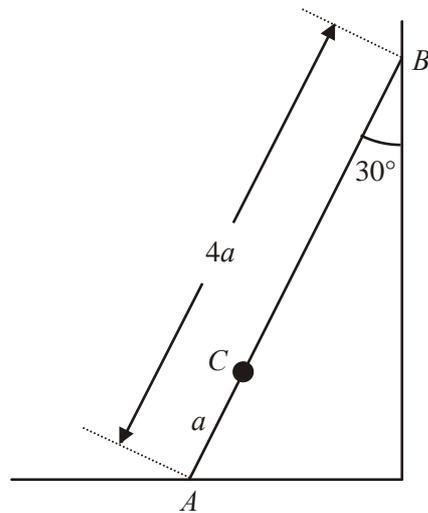
(6)

- (c) State how you have used the modelling assumption that Reece is a particle.

(1)

(Total 10 marks)

4.

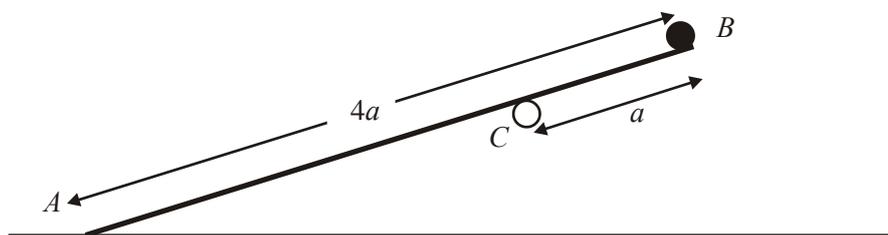


A ladder AB , of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C , where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in the diagram above.

Find the coefficient of friction between the ladder and the ground.

(Total 10 marks)

5.



A wooden plank AB has mass $4m$ and length $4a$. The end A of the plank lies on rough horizontal ground. A small stone of mass m is attached to the plank at B . The plank is resting on a small smooth horizontal peg C , where $BC = a$, as shown in the figure above. The plank is in equilibrium making an angle α with the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the plank and the ground is μ . The plank is modelled as a uniform rod lying in a vertical plane perpendicular to the peg, and the stone as a particle.

Show that

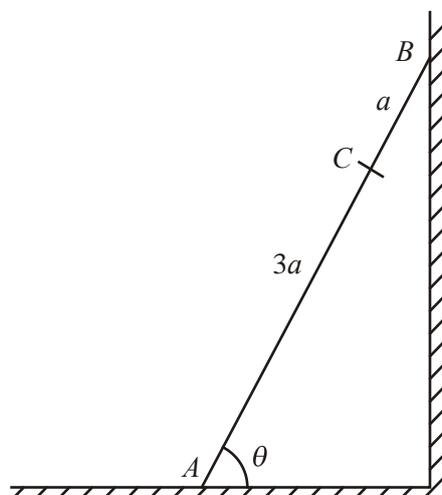
(a) the reaction of the peg on the plank has magnitude $\frac{16}{5} mg$, (3)

(b) $\mu \geq \frac{48}{61}$ (6)

(c) State how you have used the information that the peg is smooth. (1)

(Total 10 marks)

6.



A ladder AB , of weight W and length $4a$, has one end A on rough horizontal ground. The coefficient of friction between the ladder and the ground is μ . The other end B rests against a smooth vertical wall. The ladder makes an angle θ with the horizontal, where $\tan \theta = 2$. A load of weight $4W$ is placed at the point C on the ladder, where $AC = 3a$, as shown in the figure above. The ladder is modelled as a uniform rod which is in a vertical plane perpendicular to the wall. The load is modelled as a particle. Given that the system is in limiting equilibrium,

(a) show that $\mu = 0.35$.

(6)

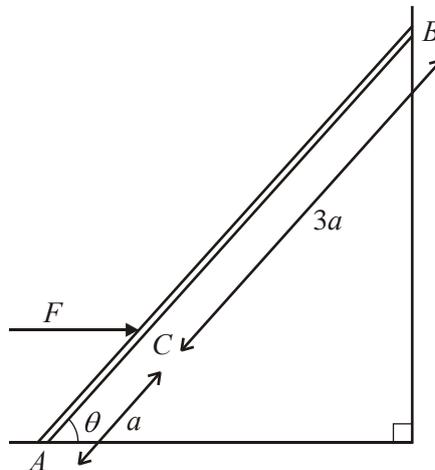
A second load of weight kW is now placed on the ladder at A . The load of weight $4W$ is removed from C and placed on the ladder at B . The ladder is modelled as a uniform rod which is in a vertical plane perpendicular to the wall. The loads are modelled as particles. Given that the ladder and the loads are in equilibrium,

(b) find the range of possible values of k .

(7)

(Total 13 marks)

7.



A uniform ladder AB has one end A on smooth horizontal ground. The other end B rests against a smooth vertical wall. The ladder is modelled as a uniform rod of mass m and length $4a$. The ladder is kept in equilibrium by a horizontal force F acting at a point C of the ladder where $AC = a$. The force F and the ladder lie in a vertical plane perpendicular to the wall. The ladder is inclined to the horizontal at an angle θ , where $\tan \theta = 2$, as shown in the diagram above.

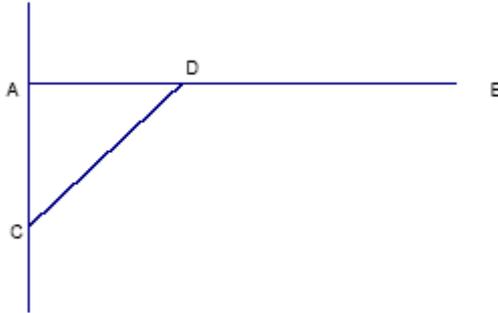
Find F in terms of m and g .

(Total 6 marks)

1. $m(B) : R \times 4 \cos \alpha = F \times 4 \sin \alpha + 20g \times 2 \cos \alpha$ M1 A2
 Use of $F = \frac{1}{2}R$ M1
 Use of correct trig ratios B1
 $R = 160\text{N}$ or 157N DM1 A1

[7]

2. (a)



Taking moments about A:

$$3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5 \quad \text{M1A1A1}$$

$$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4\text{N}) \quad \text{A1} \quad 4$$

(b) $\leftarrow \pm H = \frac{T}{\sqrt{2}} \left(= \frac{9g}{2} \approx 44.1\text{N} \right)$ B1

$$\uparrow \pm V + \frac{T}{\sqrt{2}} = 3g \left(\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7\text{N} \right) \quad \text{M1A1}$$

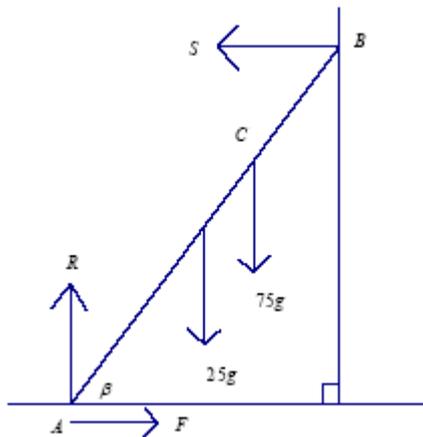
$$|R| = \sqrt{81 + 9} \times \frac{g}{2} \approx 46.5(\text{N}) \quad \text{M1A1}$$

at angle $\tan^{-1} \frac{1}{3} =$ M1A1 7
 18.4° (0.322 radians) below the line of BA
 161.6° (2.82 radians) below the line of AB
 (108.4° or 1.89 radians to upward vertical)

[11]

3. (a) $R(\uparrow) : R = 25g + 75g (= 100g)$ B1
 $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ M1
 $= 44g (= 431)$ A1 3

(b)



M(A):

$$25g \times 2\cos\beta + 75g \times 2.8\cos\beta = S \times 4\sin\beta$$

R(\leftrightarrow): $F = S$

$$176g\sin\beta = 260g\cos\beta$$

$$\beta = 56^\circ$$

M1
A2, 1, 0

M1A1

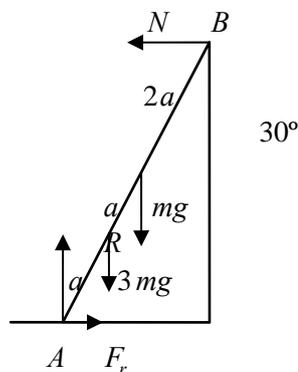
A1 6

(c) So that Reece's weight acts directly at the point C.

B1 1

[10]

4. (a)



M(A) $N \times 4a \cos 30^\circ = 3mg \times a \sin 30^\circ + mg \times 2a \sin 30^\circ$

M1 A2(1,0)

$$N = \frac{5}{4}mg \tan 30^\circ (= \frac{5}{4\sqrt{3}}mg = 7.07\dots m)$$

DM1 A1

$\rightarrow F_r = N, \uparrow R = 4mg$

B1, B1

Using $F_r = \mu R$

B1

$$\frac{5}{4\sqrt{3}}mg = \mu R \text{ for their } R$$

M1

$$\mu = \frac{5}{16\sqrt{3}}$$

awrt 0.18

A1 10

Alternative method:

$$M(B): mg \times 2a \sin 30 + 3mg \times 3a \sin 30 + F \times 4a \cos 30 = R \times 4a \sin 30$$

$$11mga \sin 30 + F \times 4a \cos 30 = R \times 4a \sin 30$$

M1A3(2,1,0)

$$\frac{11mg}{2} + F \frac{4\sqrt{3}}{2} = 2R$$

DM1A1

$$\uparrow R = 4mg,$$

B1

$$\text{Using } F_r = \mu R$$

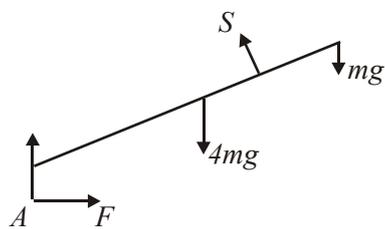
B1

$$8\mu\sqrt{3} = \frac{5}{2}, \mu = \frac{5}{16\sqrt{3}}$$

M1 A1

[10]

5. (a)



M(A):

$$S \cdot 3a = 4mg \cdot 2a \cos \alpha + mg \cdot 4a \cos \alpha$$

M1 A1

$$= \frac{48}{5} mga \Rightarrow S = \frac{16}{5} mg$$

A1 3

(b) R(\uparrow): $R + S \cos \alpha = 5mg$

M1 A1

R(\rightarrow): $F = S \sin \alpha$

M1 A1

$$F \leq \mu R \Rightarrow \mu \geq \frac{48}{61}$$

dep on both previous M's

M1 A1 6

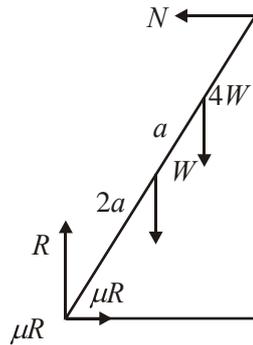
(c) Direction of S is perpendicular to plank

or No friction at the peg

B1 1

[10]

6. (a)



$\uparrow R = 5W$

$M(B): 4Wa \cos \theta + W \cdot 2a \cos \theta + \mu R 4a \sin \theta = R \cdot 4a \cos \theta$

Having enough equations & solving them for μ

$\mu = 0.35$

B1

B1

M1 A1

M1

A1

6

(b) $\uparrow S = (5 + k)W$

Use of $F = 0.35S$ or $F \leq 0.35S$

$M(B): kW4a \cos \theta + W \cdot 2a \cos \theta + F4a \sin \theta = S \cdot 4a \cos \theta$

Having enough equations & solving them for k

$k = \frac{10}{7}$

awrt 1.42

A1

$k \mid \frac{10}{7}$

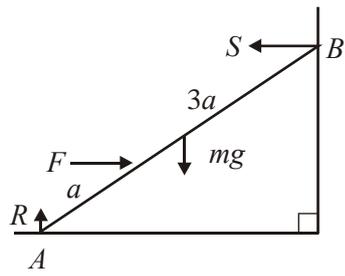
ft their k , accept $>$ and decimals

A1ft

7

[13]

7.



$$R (\rightarrow), F = S$$

B1

$$M (A)$$

$$mg \quad mg 2a \cos \theta + F a \sin \theta = S \times 4a \sin \theta$$

M1 A2

$$\text{i.e. } 2mg + 2F = 8S$$

$$F = \frac{1}{3}mg$$

M1 A1

6

[6]

1. There were very few correct solutions to this question that did not involve taking moments about B . Many candidates seemed to assume that the lack of any information about the direction of the force at B was an omission rather than a hint on how to proceed. Those candidates who started by taking moments about B usually reached the required answer without difficulty. The most common errors involved confusion between sine and cosine, and inappropriate accuracy in the final answer after using a decimal approximation for g . Alternative methods involving the force at B rarely produced a complete solution. Many candidates assumed that the direction of this force involved the angle α , thus simplifying the algebraic manipulation of their force and moment equations. Those who introduced an unknown angle usually struggled to reach a valid answer, although a handful of concise, correct solutions were seen.

2. In part (a) candidates who realised the thrust in the strut acted in the direction CD were generally successful in finding its magnitude. The overwhelming majority applied the most simple method of taking moments about A , although much longer alternative methods were often seen. A significant number, however, appeared to be confused by the use of the word thrust and many took this to be a vertical force acting at D .

For part (b) many candidates were able to find the horizontal and vertical components of the reaction at A and then the correct magnitude of this force. Most went on to find a direction but a significant number were unable to describe this direction properly. It was surprising to see relatively few diagrams and yet a diagram would have shown direction clearly.

The confusion over the direction of the thrust led to many errors in the vertical component of the reaction. Some candidates falsely assumed that the reaction at A acted vertically/horizontally. The candidates who took the force at D to be vertical were unable to complete this part as there was no horizontal force present.

3. This question was answered well, with few instances this time of the reaction at the ground or at the wall being in the wrong direction.

In (a) candidates had little problem finding the frictional force acting but then the majority gave the answer to 4 significant figures, losing the final mark.

Most candidates then went on to take moments about A or B . Errors at this stage were usually due to terms being dimensionally incorrect – often leaving out the distance in one or more terms of the moments equation. Virtually all candidates went on to find a value for β , but this was not always expressed to the required degree of accuracy.

Most candidates demonstrated some understanding of what it meant to model Reece as a particle, but few were sufficiently precise in their responses. Many mentioned mass acting at a point rather than weight and few were specific about where the weight was assumed to act.

4. The open ended nature of this question produced varied methods, but the successful candidates tended to take moments about A or B only. It was surprisingly common to see no consideration of g or even a weightless ladder, both of which carried standard penalties. The frequent omission of g was particularly disappointing at this level. A large number of candidates had mainly correct methods but made errors in manipulation, particularly in dealing with the $\sqrt{3}$; a similarly large number confused 30° with 60° (or sine with cosine). Although most candidates tried to take moments, virtually none went for the direct (and simple) option of taking moments about the point of intersection of the lines of action of the reactions at the ground and at the

wall. The better candidates were able answer this question in a couple of lines. Weaker candidates tended either to fail to take moments about any point or to get very involved taking moments about every point they thought relevant. Despite the less structured style of the question, this is a very familiar topic, so it was disappointing to find several candidates with little idea of how to tackle the problem.

5. This was probably the worst attempted question of all. Some candidates clearly knew how to deal with questions of this type and had correct forces in their diagram and were able to demonstrate an efficient use of moments equations and resolution equations to achieve the correct answers. Too many, however, were unable to put the correct forces on their diagram. The normal reaction at the peg was often acting vertically or even horizontally and all too often extra forces at B were also present. The reaction on the rod at A was often acting perpendicular to the rod instead of perpendicular to the ground. These errors and others then either led to incorrect equations or to there being too many unknowns, or both. Many managed to form the moments equation required for part (a) although those who had a vertical force at C were then unable to prove the required result. Part (b) was very poorly done. Of those who formed correct equations many assumed limiting equilibrium instead of using an inequality and these candidates were unable to score full marks as a result. A significant number attempted to use moments instead of simply resolving horizontally and vertically, making the solution much more difficult. Almost all candidates answered the final part correctly.

6. This was the most challenging question on the paper and very few scored full marks. Part (a) was generally well attempted with the printed answer undoubtedly helping. Many completely correct moments equations were seen and it was pleasing to note that few of the common errors such as missing terms, incorrect trig ratios or missing distances were made. The second part was more difficult algebraically, but nevertheless many candidates successfully found the value of k . However, it was surprising to see so many candidates, who produced a correct solution to part (a), make a complete hash of part (b). Full marks in the second part were very rare as the vast majority chose to work with $F = \mu R$ instead of $F \leq \mu R$ and produced the inequality “out of a hat” at the end. Some candidates included g in their equations despite being given that W was a weight.

7. No Report available for this question.